University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

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BEng (University of Science and Technology of China, 2012)

"Computing Optimal Designs for Regression Models via Convex Programming"

Department of Mathematics and Statistics

Tuesday August 4th, 2015
2:00 P.M.
David Strong Building
Room C130

Supervisory Committee:
Dr. Jane Ye, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Julie Zhou, Department of Mathematics and Statistics, UVic (Co-Supervisor)

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Dr. Margaret-Anne Storey, Department of Computer Science, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

Optimal design problems aim at selecting design points optimally with respect to certain statistical criteria. The research of this thesis focuses on optimal design problems with respect to A-, D- and E-optimal criteria, which minimize the trace, determinant and largest eigenvalue of the information matrix, respectively.

Semidefinite programming (SDP) is concerned with optimizing a linear objective function subject to a linear matrix being positive semidefinite. Two powerful MATLAB add-ons, SeDuMi and CVX, have been developed to solve SDP problems efficiently. In this paper, we show in detail how to formulate A- and E-optimal design problems as SDP problems and solve them by SeDuMi and CVX. This technique can be used to construct approximate A-optimal and E-optimal designs for all linear and non-linear models with discrete design spaces. The results can also provide guidance to find optimal designs on continuous design spaces. For one variable polynomial regression models, we solve the A- and E- optimal designs on the continuous design space by using a two-stage procedure. In the first stage we find the optimal moments by casting it as an SDP problem and in the second stage we extract the optimal designs from the optimal moments obtained from the first stage.

Unlike E- and A-optimal design problems, the objective function of D-optimal design problem is nonlinear. So D-optimal design problems cannot be reformulated as an SDP. However, it can be cast as a convex problem and solved by an interior point method. In this thesis we give details on how to use the interior point method to solve D-optimal design problems.

Finally several numerical examples for A-, D-, and E-optimal designs along with the MATLAB codes are presented.

Keywords: A-optimality, approximate design, convex programming, CVX, D-optimality, E-optimality, interior point method, SeDuMi, semidi_nite programming.